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ON THE RATE OF CONVERGENCE IN SOME MEAN MARTINGALE CENTRAL LIMIT THEOREMS

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Abstract: Let $(X_k; 1 \le k \le n)$ be a sequence of martingale differences with respect to σ-fields $\mathcal{F}_0 \subset \mathcal{F}_1 \subset \ldots \subset \mathcal{F}_n$, where the variance of X_k may be finite or infinite. The aim of this article is to establish the rate of convergence in the mean central limit theorems for the sum $S_n = X_1 + \ldots + X_n$ by uniting the method of Bolthausen [2], Haeusler [8] and the result of Röllin [10].

Key words: infinite variance; the central limit theorem; random variables; convergence rate; martingale difference.

1. Introduction

Let $(X_k; 1 \le k \le n)$ be a sequence of square integrable martingale difference with respect to σ-fields $\mathcal{F}_0 \subset \mathcal{F}_1 \subset \ldots \subset \mathcal{F}_n$, that is, suppose that X_k is measurable with respect to $\mathcal{F}_{k-1}$ with $E(X_k^2) < \infty$ and $E(X_k \mid \mathcal{F}_{k-1}) = 0$ for all $1 \le k \le n$. Denote F_n and Φ be distribution functions of $S_n = X_1 + \ldots + X_n$ and standard normal $N(0,1)$, respectively. Assume that $E(|X_i|^2) < \infty$ for all $i \ge 1$, according to Theorem 3.2 of Agnew [1], "the conditional Linderberg condition"

$$\sum_{i=1}^{n} E(X_i^2 I(|X_i| > \varepsilon) \mid \mathcal{F}_{i-1}) \xrightarrow{P} 0 \text{ as } n \to \infty$$

for each $\varepsilon > 0$ and "the conditional normalizing condition"

$$\sum_{i=1}^{n} E(X_i^2 \mid \mathcal{F}_{i-1}) \xrightarrow{P} 1 \text{ as } n \to \infty,$$

together imply the mean central limit theorem

$$\|F_n - \Phi\|_1 = \int_{\mathbb{R}} |F_n(x) - \Phi(x)|\, dx \to 0 \text{ as } n \to \infty.$$

Obviously, "the conditional Linderberg condition" is satisfied if for some $\delta > 0$, "the Liapounov condition of order $2 + 2\delta$"

$$\sum_{i=1}^{n} E(|X_i|^{2+2\delta}) \longrightarrow 0 \text{ as } n \to \infty \text{ holds.}$$

The rate of convergence in the central limit theorem was studied by many authors. In 1982, Bolthausen [2] established the rate of convergence in central limit theorems for bounded martingale difference sequences in the Kolmogorov metric (d_K). Then, Mourrat [9] generalized this result. Haeusle [7, 8], El Machkouri and Ouchti [6] extended the results of Bolthausen [2] to unbounded martingale difference sequences.

The rate at which $\|F_n - \Phi\|_1$ converges to zero has studied by Dung et. al. [4], Röllin [10] and other authors under the assumption of bounded third order moments. Dung and Son [4] established the rate of convergence in the central limit theorem for arrays of martingale difference random vectors in the bounded Lipschitz metric. In this article, we unite the method of Bolthausen [2], Haeusler [8] and the result of Röllin [10] to establish the convergence rate to zero of $\|F_n - \Phi\|_1$ for general martingale differences sequences under "the Liapounov condition of order $2 + 2\delta$" assumption for some $\delta > 0$. The convergence rate to

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zero of $\|F_n - \Phi\|_1$ for sequences of general martingale differences with infinite variances is also studied.

2. Results

The Wasserstein distance between two distributions F and G on real line is defined as

$$d_W(F,G) = \sup_{f \in \Lambda_1} |E(f(X)) - E(f(Y))|,$$

where Λ_1 is the set of 1-Lipschitzian functions from $\mathbb{R}$ to $\mathbb{R}$, i.e.

$$\Lambda_1 = \{f : \mathbb{R} \to \mathbb{R} : |f(x) - f(y)| \le |x - y|\}.$$

According to the Kantorovich-Rubinstein theorem (see, e.g., [3], Theorem 11.8.2) we have that

$$d_W(F,G) = \|F - G\|_1.$$

It is not difficult to see that for any random variable X,

$$d_K(F_X,\Phi) \le \frac{2}{(2\pi)^{1/4}}\sqrt{d_W(F_X,\Phi)}.$$

Let $(X_k; 1 \le k \le n)$ be a sequence of square integrable martingale difference with respect to σ-fields $\mathcal{F}_0 \subset \mathcal{F}_1 \subset \ldots \subset \mathcal{F}_n$.

Without loss of generality we may assume that $\sigma_n = E(X_1^2) + \ldots + E(X_n^2) = 1$. For $1 \le k \le n$, we define

$$\sigma_k^2 = E(X_k^2 \mid \mathcal{F}_{k-1}), \quad V_n^2 = \sum_{i=1}^n \sigma_i^2,$$

$$\rho_{k+1}^2 = V_n^2 - V_k^2 = \sum_{i=k+1}^n \sigma_i^2,$$

where $\sum_{i=k+1}^n \sigma_i^2$ is defined to be zero if $k+1 > n$. Set

$$L_{n,\delta} = \sum_{i=1}^n E(|X_i|^{2+2\delta}),$$

$$N_{n,\delta} = E\left(\left|V_n^2 - 1\right|^{1+\delta}\right)$$

for some $\delta > 0$.

Put $S_n = X_1 + X_2 + \ldots + X_n$.

For $x > 0$ we denote $\log(x) = \max\{1, \ln(x)\}$, where $\ln(x)$ is the natural logarithm function.

Theorem 1. *Assume that $V_n = 1$ a.s. Then*

$$d_W(\mathcal{L}(S_n), N(0,1)) \le C L_{n,\delta}^{\frac{1}{2(1+\delta)}}(1 + |\log L_{n,\delta}|)$$

whenever $L_{n,\delta} \le 1$.

Proof.

For fixed $0 < \beta \le 1$, set

$$X_i' = X_i I(|X_i| \le \beta^{1/2}/2) - E(X_i I(|X_i| \le \beta^{1/2}/2) \mid \mathcal{F}_{i-1}),$$

$$S_n' = \sum_{i=1}^n X_i'.$$

For any function $f \in \Lambda_1$, we have

$$\begin{aligned}|E(f(S_n)) - E(f(N(0,1)))| &\le |E(f(S_n')) - E(f(N(0,1)))| \\ &\quad + \sum_{i=1}^n E(|X_i| I(|X_i| > \beta^{1/2}/2)).\end{aligned}$$

It is obvious that

$$\sum_{i=1}^n E(|X_i| I(|X_i| > \beta^{1/2}/2)) \le \frac{C}{\beta^{(1/2+\delta)}} L_{n,\delta}.$$

Then

$$|E(f(S_n)) - E(f(N(0,1)))| \le$$
$$|E(f(S_n')) - E(f(N(0,1)))| + \frac{C}{\beta^{(1/2+\delta)}} L_{n,\delta}.$$

Let X_{n+1}', $X_{n+2}', \ldots$ be independent random variables with

$$P(X_i' = -\beta^{1/2}) = P(X_i' = \beta^{1/2}) = 1/2$$

for all $i \ge n+1$, which are independent of $\mathcal{F}_n$. For each $i \ge n+1$, set $\mathcal{F}_i = \sigma(\mathcal{F}_n, X_{n+1}', \ldots, X_i')$. It is clear that the random variable

$$\tau = \max\{k : \sum_{i=1}^k \sigma_i^2(X_i') \le 1\}$$

is a stopping time with respect to $(\mathcal{F}_i; i \ge 1)$ and $n \le \tau \le n + [b^{-1}]$, where $[b^{-1}]$ denotes the integer part of b^{-1}. For $i = 1,\ldots,k = n + [b^{-1}] + 1$, set

$$Z_i = X_i' I(i \le \tau) + \left(b^{-1}\left(1 - \sum_{i=1}^{\tau} \sigma_i^2(X_i')\right)\right)^{1/2} X_i' I(i = \tau + 1).$$

Thus, $(Z_i; 1 \le i \le k)$ is a sequence of martingale differences adapted to the filtration $\mathcal{F}_0,\ldots,\mathcal{F}_k$. Writing from now on

$$\sigma_i'^2 = \sigma_i^2(Z_i), \quad S_k'' = Z_1 + \ldots + Z_k.$$

We have that

$$\sum_{i=1}^{k} \sigma_i'^2 = 1, \quad |Z_i| \le b^{1/2} \text{ for } i = 1,\ldots,k,$$

and

$$E(|S_n' - S_k''|) \le C\left(\sum_{i=n+1}^{k} E(|Z_i|^2)\right)^{1/2} \le C b^{-\delta/2} L_{n,\delta}^{1/2}.$$

Therefore,

$$|E(f(S_n)) - E(f(N(0,1)))| \le |E(f(S_k'')) - E(f(N(0,1)))| + C\left(\frac{1}{b^{(1/2+\delta)}} L_{n,\delta} + b^{-\delta/2} L_{n,\delta}^{1/2}\right). \quad (0.1)$$

Put

$$V_i'^2 = \sum_{j=1}^{i} \sigma_j'^2, \quad r_i'^2 = 1 - V_{i-1}'^2 = \sum_{j=i}^{k} \sigma_j'^2.$$

Now define the sequence of stopping times

$$\tau_0 = 0,$$

$$\tau_j = \sup\{m \ge 0 : V_m'^2 \le jb\} \text{ for } 1 \le j \le [b^{-1}],$$

$$\tau_{[b^{-1}]+1} = k.$$

For $\tau_{j-1} < i \le \tau_j$, we have

$$r_i'^2 = 1 - V_{i-1}'^2 \ge 1 - jb.$$

Thus, it follows from Theorem 2.1 of Röllin [10] with $a = (2b)^{1/2}$ that

$$|E(f(S_k'')) - E(f(N(0,1)))|$$

$$\le 3\sum_{i=1}^{k} E\left(\frac{|Z_i|^3}{r_i'^2 + 2b}\right) + 2\sqrt{2}b^{1/2}$$

$$= 3\sum_{j=1}^{[b^{-1}]+1} \sum_{i=\tau_{j-1}+1}^{\tau_j} E\left(\frac{|Z_i|^3}{r_i'^2 + 2b}\right) + 2\sqrt{2}b^{1/2}$$

$$\le 3\sum_{j=1}^{[b^{-1}]+1} \frac{b^{1/2}}{1 - jb + 2b} \sum_{i=\tau_{j-1}+1}^{\tau_j} E(|Z_i|^2) + 2\sqrt{2}b^{1/2}$$

$$= 3\sum_{j=1}^{[b^{-1}]+1} \frac{b^{1/2}}{1 - jb + 2b} E\left(\sum_{i=\tau_{j-1}+1}^{\tau_j} E(|Z_i|^2 | \mathcal{F}_{i-1})\right) + 2\sqrt{2}b^{1/2}$$

$$= 3\sum_{j=1}^{[b^{-1}]+1} \frac{b^{1/2}}{1 - jb + 2b} E\left(V_{\tau_j}'^2 - V_{\tau_{j-1}}'^2\right) + 2\sqrt{2}b^{1/2}$$

$$\le 3\sum_{j=1}^{[b^{-1}]+1} \frac{2b^{3/2}}{1 - jb + 2b} + 2\sqrt{2}b^{1/2}$$

$$\le 6b^{1/2} \sum_{j=1}^{[b^{-1}]+1} \frac{1}{j} + 2\sqrt{2}b^{1/2}$$

$$\le C(b^{1/2}|\log b| + 2\sqrt{2}b^{1/2}.$$

$$\le 3\sum_{j=1}^{[b^{-1}]+1} \frac{2b^{3/2}}{1 - jb + 2b} + 2\sqrt{2}b^{1/2}$$

Combining this with (1.1) we obtain

$$|E(f(S_n)) - E(f(N(0,1))|$$

$$\le C(b^{1/2}|\log b|$$

$$+ \frac{1}{b^{(1/2+\delta)}} L_{n,\delta} + b^{-\delta/2} L_{n,\delta}^{1/2} + b^{1/2}).$$

Put $b = L_{n,\delta}^{1/(1+\delta)}$ we get

$$|E(f(S_n)) - E(f(N(0,1)))| \le CL_{n,\delta}^{\frac{1}{2(1+\delta)}}(1+|\log L_{n,\delta}|).$$

Thus,

$$d_W(\mathcal{L}(S_n),N(0,1)) \le CL_{n,\delta}^{\frac{1}{2(1+\delta)}}(1+|\log L_{n,\delta}|)$$

which completes the proof.

If the assumption of $V_n = 1$ a.s. is removed, we obtain the following theorem.

Theorem 2.

$$d_W(\mathcal{L}(S_n),N(0,1)) \le C(2L_{n,\delta}+N_{n,\delta})^{\frac{1}{2(1+\delta)}}(1+|\log(2L_{n,\delta}+N_{n,\delta})|).$$

whenever $2L_{n,\delta}+N_{n,\delta} \le 1$.

Proof.

We define the stopping time τ by

$$\tau = \max\{k : \sum_{i=1}^{k} \sigma_i^2 \le 1\},$$

put

$$\tilde X_i = X_i I(i \le \tau) \text{ for } 1 \le i \le n$$
$$\text{and } \tilde X_{n+1} = (1-\sum_{i=1}^{\tau}\sigma_i^2)^{1/2} Y,$$

where the random variable Y is independent of $\mathcal{F}_n$ with $P(Y=1)=P(Y=-1)=1/2$. Then, $(\tilde X_i; 1\le i\le n+1)$ is a sequence of martingale differences satisfying the assumptions of Theorem 1, so that

$$d_W(\mathcal{L}(\tilde S_n),N(0,1)) \le C\tilde L_{n,\delta}^{\frac{1}{2(1+\delta)}}(1+|\log \tilde L_{n,\delta}|),$$

where

$$\tilde S_n = \sum_{i=1}^{n+1}\tilde X_i \text{ and } \tilde L_{n,\delta} = \sum_{i=1}^{n+1} E(|\tilde X_i|^{2+2\delta}).$$

But

$$\begin{aligned}
\tilde L_{n,\delta} &= \sum_{i=1}^{n+1} E(|\tilde X_i|^{2+2\delta}) \le L_{n,\delta} + E(|\tilde X_{n+1}|^{2+2\delta}) \\
&\le L_{n,\delta} + E(|1-\sum_{i=1}^{\tau}\sigma_i^2|^{1+\delta} I(\tau = n)) \\
&\quad + E(|1-\sum_{i=1}^{\tau}\sigma_i^2|^{1+\delta} I(\tau < n)) \\
&\le L_{n,\delta} + N_{n,\delta} + E(\max_{1\le i\le n}\sigma_i^{2+2\delta}) \\
&\le 2L_{n,\delta} + N_{n,\delta}.
\end{aligned}$$

On the other hand,

$$E(|S_n - \tilde S_n|^{2+2\delta}) \le C\left(E(|\sum_{i=\tau+1}^{n} X_i|^{2+2\delta}) + E(|\tilde X_{n+1}|^{2+2\delta})\right)$$

$$E(|\tilde X_{n+1}|^{2+2\delta}) \le L_{n,\delta} + N_{n,\delta},$$

and

$$\begin{aligned}
&E(|\sum_{i=\tau+1}^{n} X_i|^{2+2\delta}) \\
&\le C\left(E(|\sum_{i=\tau+1}^{n}\sigma_i^2|^{1+\delta}) + E(\max_{\tau+1\le i\le n}|X_i|^{2+2\delta})\right) \\
&\le C\left(E(|1-\sum_{i=1}^{n}\sigma_i^2|^{1+\delta}) + E(|1-\sum_{i=1}^{\tau}\sigma_i^2|^{1+\delta})\right) \\
&\quad + CL_{n,\delta} \\
&\le C\left(2L_{n,\delta}+N_{n,\delta}\right)
\end{aligned}$$

which imply that

$$E(|S_n - \tilde S_n|^{2+2\delta}) \le C(2L_{n,\delta}+N_{n,\delta}).$$

Thus,

$$\begin{aligned}
&d_W(\mathcal{L}(S_n),N(0,1)) \\
&\le d_W(\mathcal{L}(\tilde S_n),N(0,1)) + E(|S_n - \tilde S_n|) \\
&\le d_W(\mathcal{L}(\tilde S_n),N(0,1)) + E(|S_n - \tilde S_n|^{2+2\delta})^{\frac{1}{2(1+\delta)}} \\
&\le C(2L_{n,\delta}+N_{n,\delta})^{\frac{1}{2(1+\delta)}}(1+\log(2L_{n,\delta}+N_{n,\delta})).
\end{aligned}$$

Corollary. *Let* $(X_n; n\ge 1)$ *be a stationary sequence of martingale difference with respect to the* σ*-fields* $\mathcal{F}_n = \sigma(X_i : i \le n)$. *Suppose that* $E(|X_1|^{2+2\delta}) < \infty$ *for any* $\delta > 0$ *and*

$E(X_1^2 \mid \mathcal{F}_0) = E(X_1^2) = \sigma^2 > 0$. *Then, there exists a positive constant* C *depending only on* δ, *we infer*

$$\left\| F_{\frac{\sum_{i=1}^{n} X_i}{\sigma\sqrt{n}}} - \Phi \right\|_1 \le C.n^{-\delta/(2+2\delta)} \ln(n)$$

for large enough n.

Proof. Set $Y_{n,i} = \frac{\sigma^{-1}}{\sqrt{n}} X_i$ for $1 \le i \le n$, then $\sigma_n^2 = E(Y_n^2 \mid \mathcal{F}_{n-1}) = \frac{1}{n}$. We see that $(Y_n; n \ge 1)$ is a martingale difference sequence with respect to the σ-fields $(\mathcal{F}_n; n \ge 1)$.

$$\sum_{i=1}^{n} \sigma_{n,i}^2 = 1,$$

$$L_{n,\delta} = \sum_{i=1}^{n} E\left(\|Y_i\|^{2+2\delta}\right) \le \frac{E\left(\|X_1\|^{2+2\delta}\right)}{\sigma^{2+2\delta} n^{\delta}},$$

and $S_n = \sum_{i=1}^{n} Y_i = \frac{\sum_{i=1}^{n} X_i}{\sigma\sqrt{n}}$.

By Theorem 1, we have

$$\left\|F_{S_n} - \Phi\right\|_1 \le C(L_{n,2\delta})^{1/(2+2\delta)}(1 + |\ln(L_{n,2\delta})|) \le Cn^{-\delta/(2+2\delta)} \ln(n).$$

For the next theorem, we establish the convergence rate to zero of $\|F_n - \Phi\|_1$ for sequences of general martingale differences with infinite variances. Suppose that $(X_i; 1 \le i \le n)$ is a sequence of stationary martingale differences with infinite variances. Assume that $h(x) = E(X_1^2 I(|X_1| \le x))$ is a slowly varying function at infinity. Put

$$h_n = \inf\{x \ge 1 : x^{-2} h(x) \le n^{-1}\},$$

$$\underline{\sigma}_i^2 = Var(X_i I(|X_i| \le h_n) - E(X_i I(|X_i| \le h_n) \mid \mathcal{F}_{i-1})),$$

$$a_n^2 = \sum_{i=1}^{n} \underline{\sigma}_i^2,$$

$$\tilde{V}_n = \frac{1}{a_n^2} \sum_{j=1}^{n} \left\{E(((X_i I(|X_i| \le h_n) - E(X_i I(|X_i| \le h_n) \mid \mathcal{F}_{i-1}))^2 \mid \mathcal{F}_{i-1})\right\}.$$

$$K_{n,\delta} = 2^{2+2\delta} \left(\frac{h_n}{a_n}\right)^{2+2\delta} \frac{E(|X_1|^{2+2\delta} I(|X_1| \le h_n))}{h_n^{2\delta} h(h_n)},$$

$$Q_n = \frac{h_n^2 E(|X_1| I(|X_1| > h_n))}{a_n h(h_n)},$$

and

$$M_{n,\delta} = E\left\|\tilde{V}_n - 1\right\|^{1+\delta}.$$

Noting that if $h_n = O(a_n)$ then we get $K_{n,\delta} \to 0$ and $Q_n \to 0$ as $n \to \infty$. We have the following theorem.

Theorem 3. *Suppose that* $(X_i; 1 \le i \le n)$ *is a sequence of stationary martingale differences such that* $h(x) = E(X_1^2 I(|X_1| \le x))$ *is a slowly varying function at infinity. Then*

$$d_W(\mathcal{L}(S_n / a_n), N(0,1)) \le C\left((2K_{n,\delta} + M_{n,\delta})^{\frac{1}{2(1+\delta)}}(1 + |\log(2K_{n,\delta} + M_{n,\delta})|)\right) + Q_n.$$

whenever $2K_{n,\delta} + M_{n,\delta} \le 1$.

Proof.

Set

$$\tilde{X}_i = a_n^{-1}[X_i I(|X_i| \le h_n) - E(X_i I(|X_i| \le h_n \mid \mathcal{F}_{i-1})],$$

$\tilde{S}_n = \sum_{i=1}^{n} \tilde{X}_i$. Then,

$$d_W(\mathcal{L}(S_n / a_n), N(0,1)) \le d_W(\mathcal{L}(\tilde{S}_n), N(0,1)) + \frac{nE(|X_1| I(|X_1| > h_n))}{a_n}. \quad (0.2)$$

$$d_W(\mathcal{L}(\tilde{S}_n), N(0,1))$$
$$\le C(2\tilde{L}_{n,\delta} + \tilde{N}_{n,\delta})^{\frac{1}{2(1+\delta)}}(1+ |\log(2\tilde{L}_{n,\delta} + \tilde{N}_{n,\delta}|),$$

where

$$\tilde{L}_{n,\delta} = \sum_{i=1}^{n} E(|\tilde{X}_i|^{2+2\delta})$$
$$\le 2^{2+2\delta}\frac{nE(|X_1|^{2+2\delta} I(|X_1| \le h_n))}{a_n^{2+2\delta}}$$
$$\le 2^{2+2\delta}\frac{h_n^2 E(|X_1|^{2+2\delta} I(|X_1| \le h_n))}{a_n^{2+2\delta} h(h_n)}$$
$$= K_{n,\delta},$$

and where

$$\tilde{N}_{n,\delta} = E(\|\tilde{V}_n - 1\|^{1+\delta}) = M_{n,\delta}$$

Thus,

$$d_W(\mathcal{L}(\tilde{S}_n), N(0,1))$$
$$\le C(2K_{n,\delta} + M_{n,\delta})^{\frac{1}{2(1+\delta)}}(1+ |\log(2K_{n,\delta} + M_{n,\delta})|)$$

On the other hand

$$\frac{nE(|X_1| I(|X_1| > h_n))}{a_n} \le \frac{h_n^2 E(|X_1| I(|X_1| > h_n))}{a_n h(h_n)} = Q_n. \quad (0.4)$$

Combining (1.2), (1.3) and (1.4) yield the conclusion of Theorem 3.

3. Conclusion

Through this article, we have obtained some results of the convergence rate in the mean central limit theorems for sequences of general martingale differences with finite or infinite variances.

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VỀ TỐC ĐỘ HỘI TỤ TRONG MỘT SỐ ĐỊNH LÍ GIỚI HẠN TRUNG TÂM THEO TRUNG BÌNH

Tóm tắt: Cho $(X_k; 1 \le k \le n)$ là dãy hiệu martingale tương thích với dãy σ - đại số $\mathcal{F}_0 \subset \mathcal{F}_1 \subset ... \subset \mathcal{F}_n$, trong đó phương sai của biến ngẫu nhiên X_k có thể hữu hạn hoặc vô hạn. Mục đích của bài báo này là thiết lập tốc độ hội tụ trong định lí giới hạn trung tâm theo trung bình cho tổng $S_n = X_1 + ... + X_n$ bằng phương pháp của Bolthausen [2], Haeusler [8] kết hợp với kết quả của Röllin [10].

Từ khóa: phương sai vô hạn; định lí giới hạn trung tâm; tốc độ hội tụ; biến ngẫu nhiên; hiệu martingale.